Number Systems

Common Number Systems

System	***Base***	***Symbols***	***Used by humans?***	***Used in PLC?***
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	***Binary***	***Octal***	***Hexa-decimal***
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	*Binary*	*Octal*	*Hexa-decimal*
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

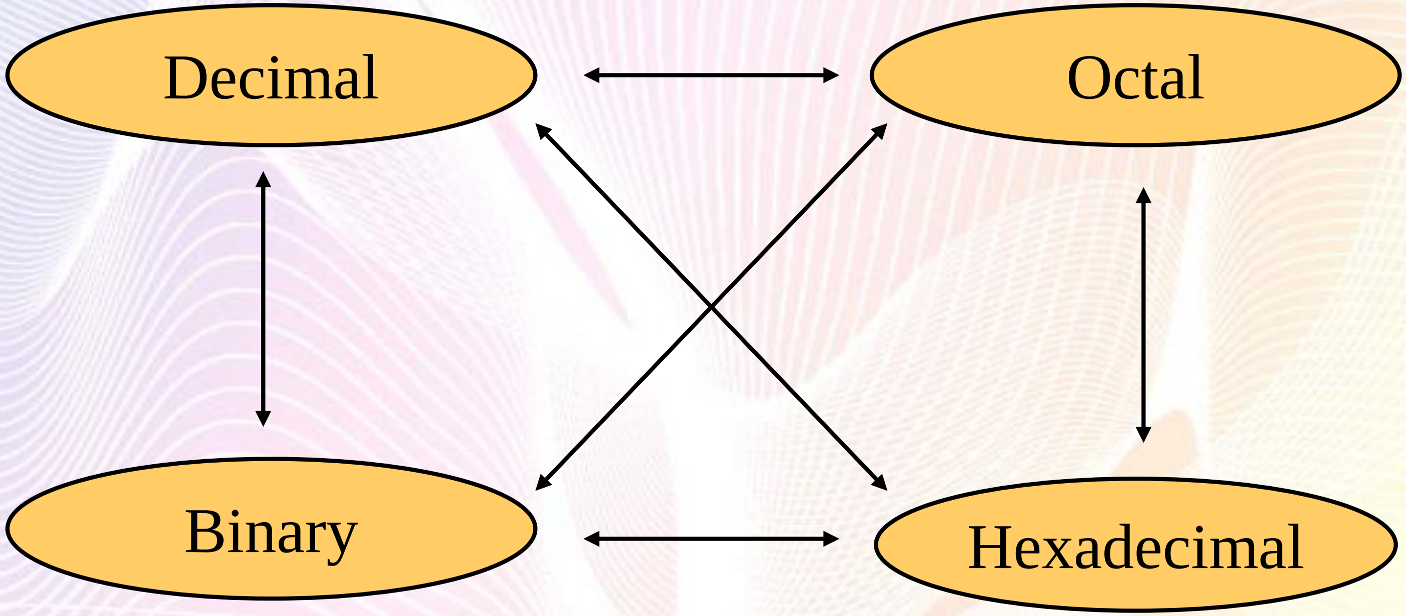
Quantities/Counting (3 of 3)

Decimal	*Binary*	*Octal*	*Hexa-decimal*
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

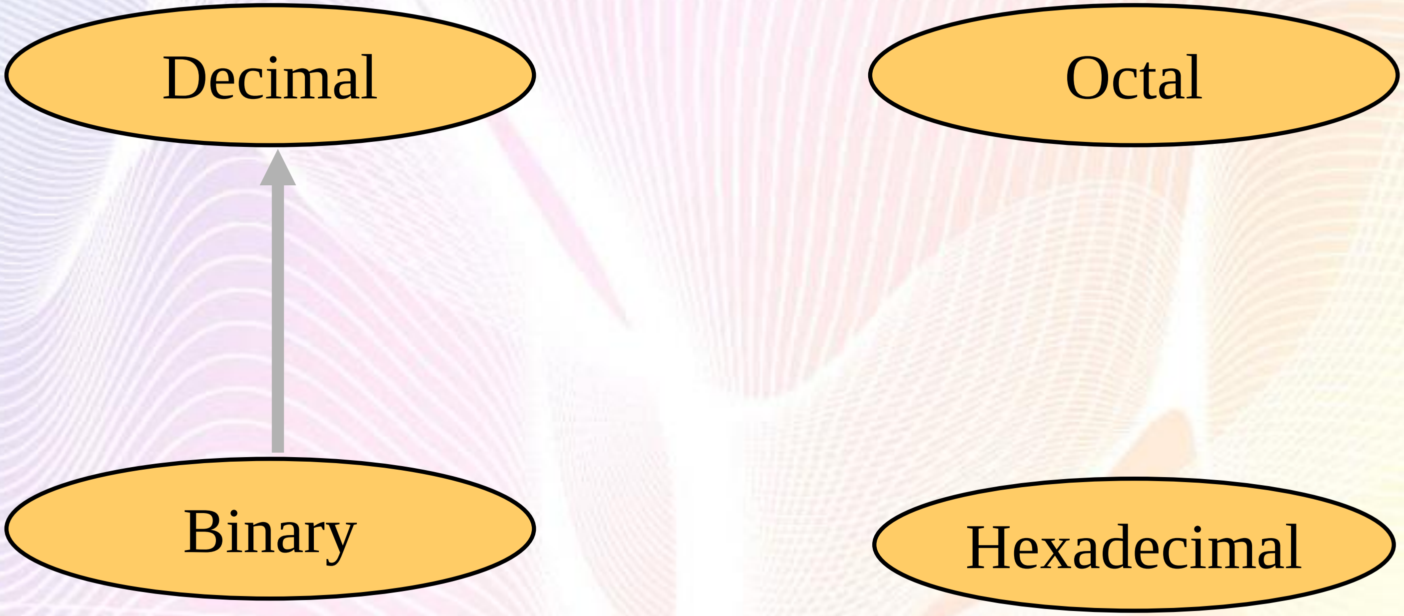
Conversion Among Bases

- The possibilities:



Conversion Among Bases

- The possibilities:



Binary to Decimal

- Technique
  - Multiply each bit by 2^n, where *n* is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

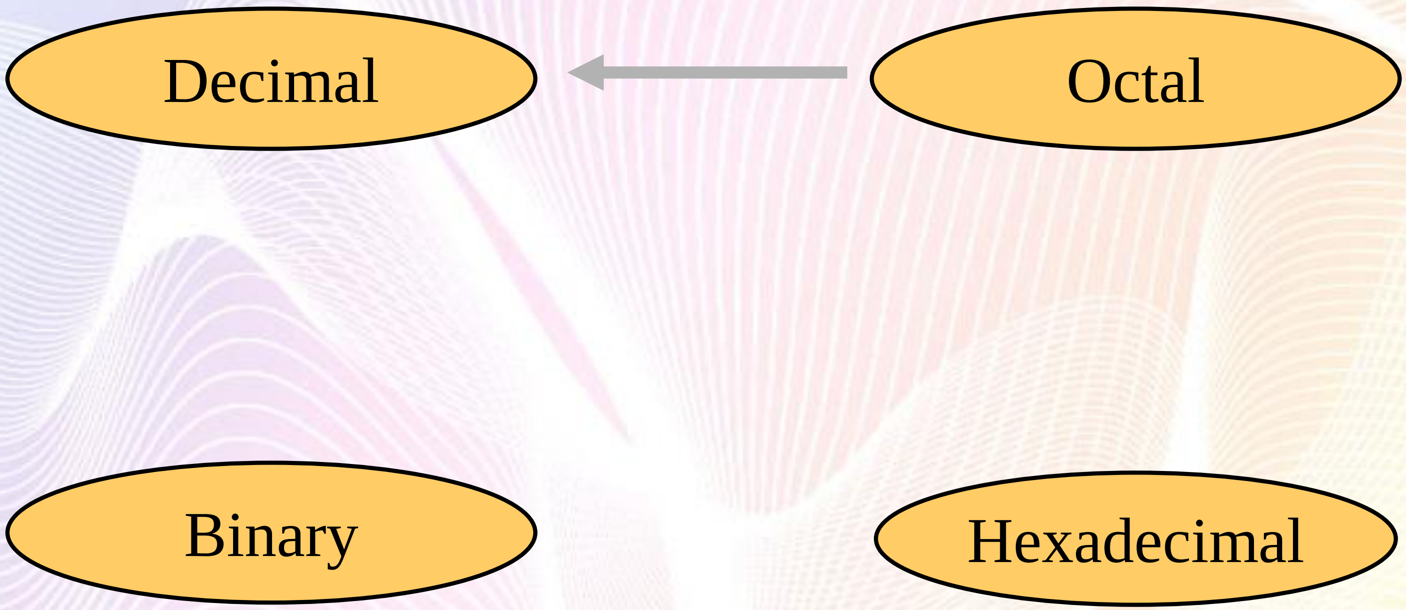
Bit “0”

101011_2 =>

$1 \times 2^0 =$ 1
$1 \times 2^1 =$ 2
$0 \times 2^2 =$ 0
$1 \times 2^3 =$ 8
$0 \times 2^4 =$ 0
$1 \times 2^5 =$ 32

43_{10}

Octal to Decimal



Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where *n* is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

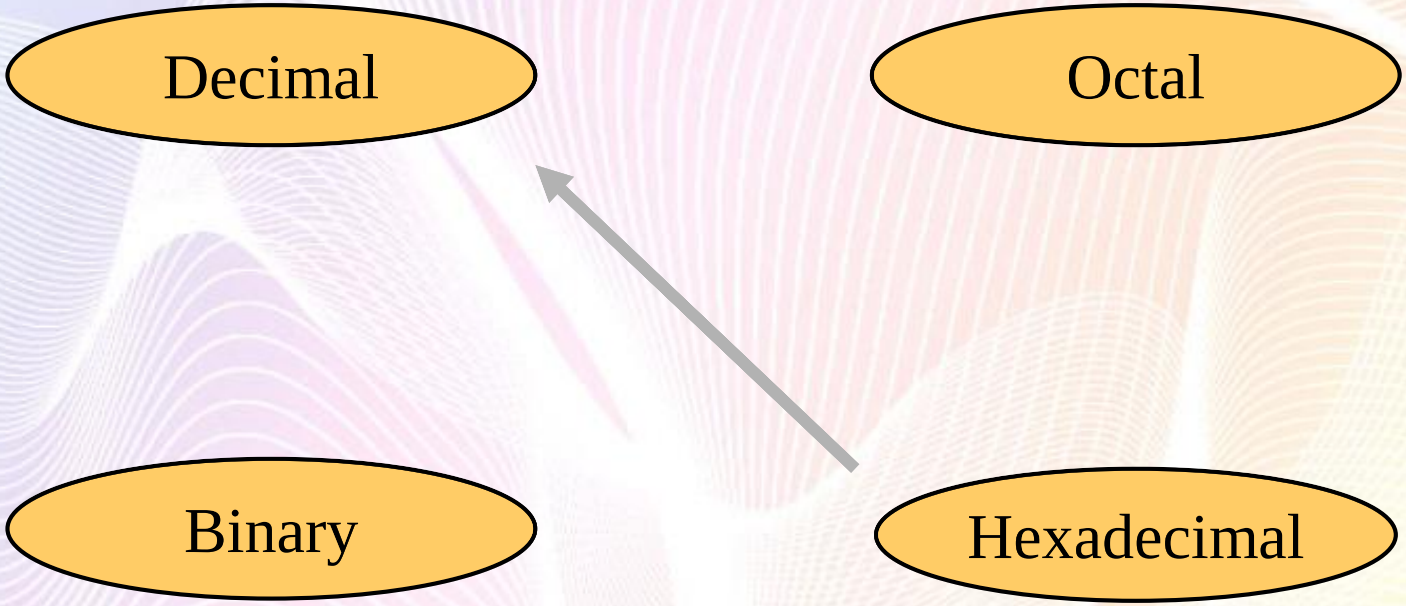
Example

Bit “0”

724_8 =>

$4 \times 8^0 =$	4
$2 \times 8^1 =$	16
$7 \times 8^2 =$	448
	468_{10}

Hexadecimal to Decimal



Hexadecimal to Decimal

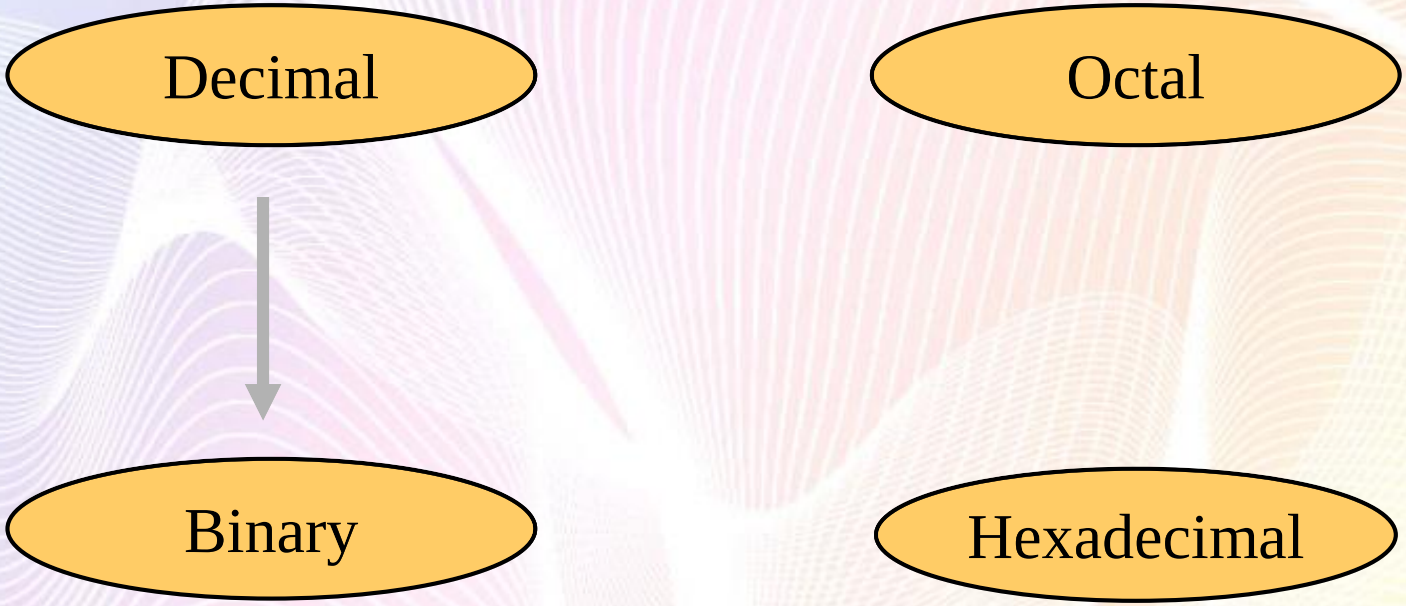
- Technique
  - Multiply each bit by 16^n, where *n* is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$ABC_{16} \Rightarrow$

$$\begin{aligned} C \times 16^0 &= 12 \times 1 = 12 \\ B \times 16^1 &= 11 \times 16 = 176 \\ A \times 16^2 &= 10 \times 256 = 2560 \\ \hline & \quad 2748_{10} \end{aligned}$$

Decimal to Binary



Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder

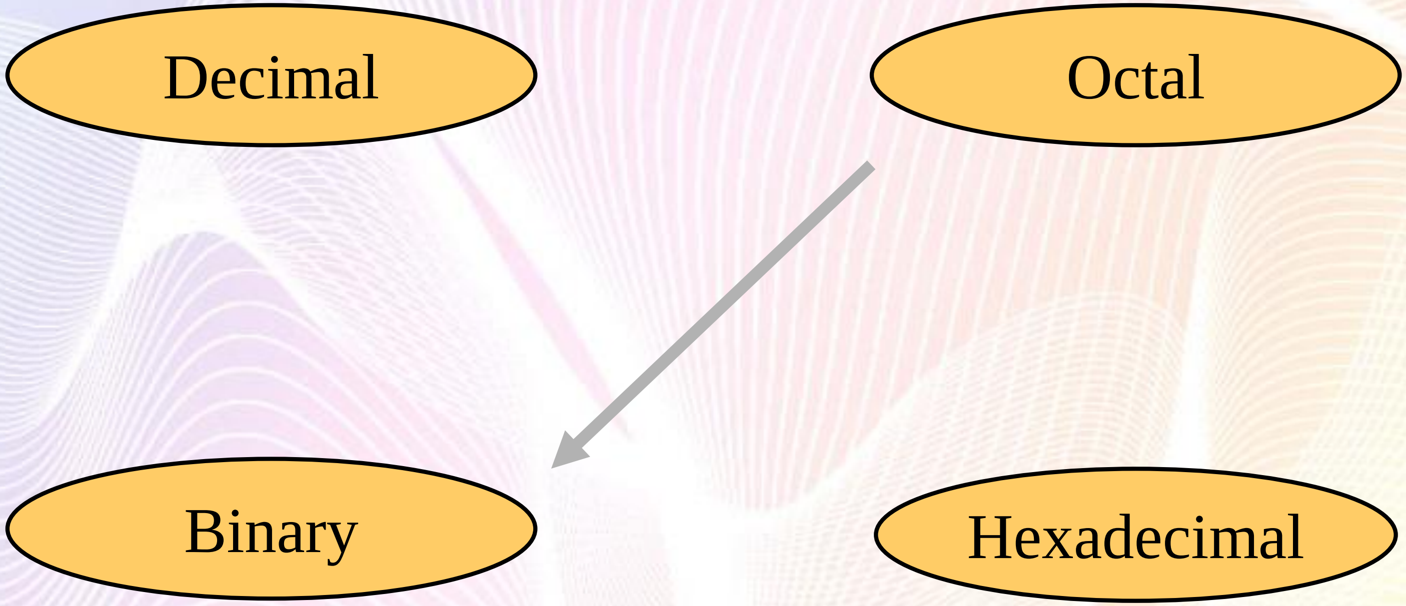
Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Octal to Binary



Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

Example

$705_8 = ?_2$

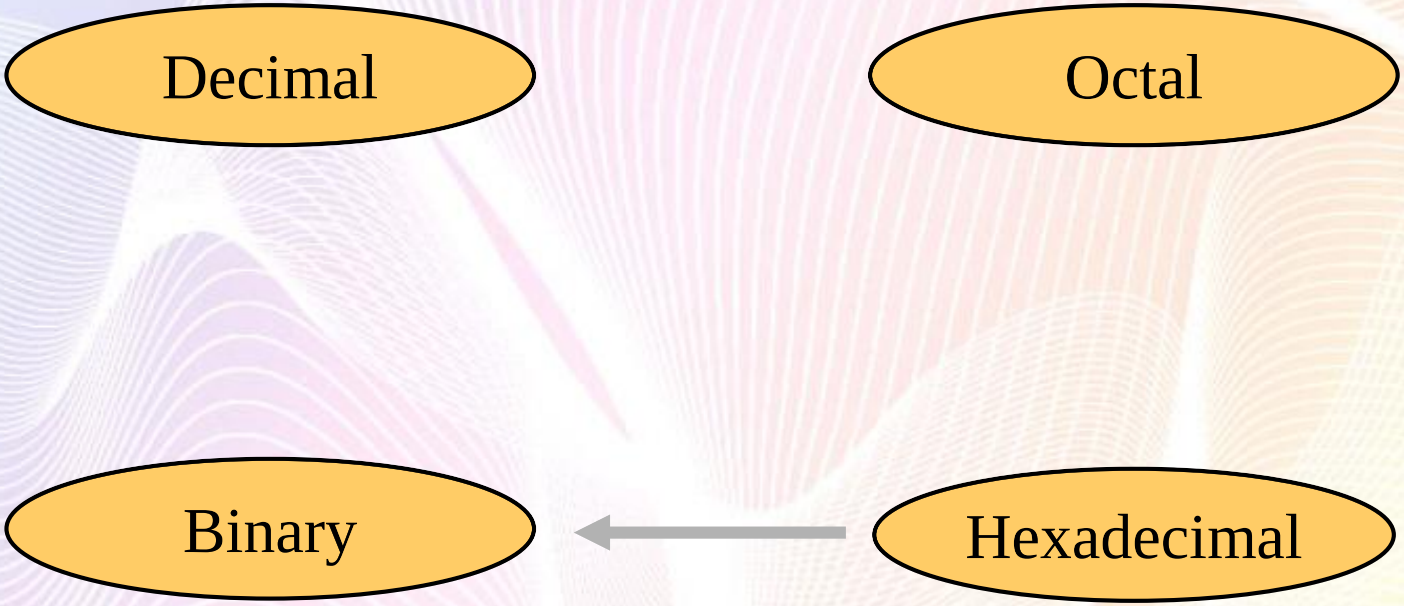
7 0 5

↓ ↓ ↓

111 000 101

$705_8 = 111000101_2$

Hexadecimal to Binary



Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

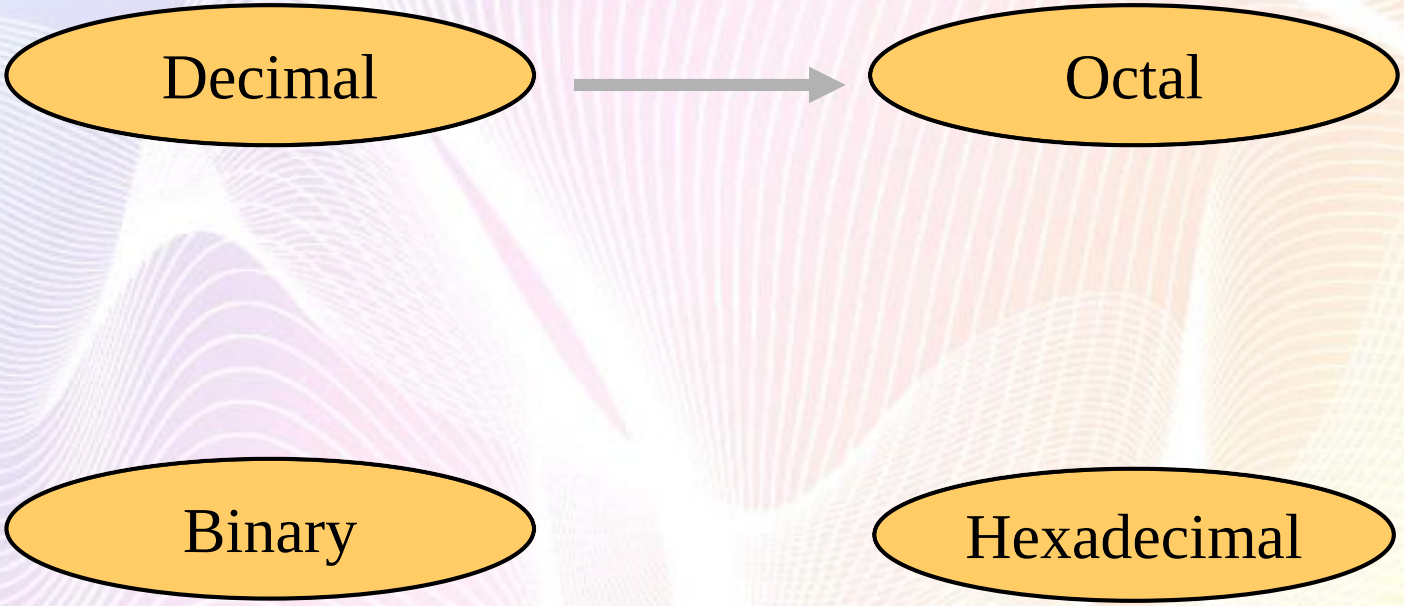
Example

$10AF_{16} = ?_2$

1	0	A	F
↓	↓	↓	↓
0001	0000	1010	1111

$10AF_{16} = 0001000010101111_2$

Decimal to Octal



Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

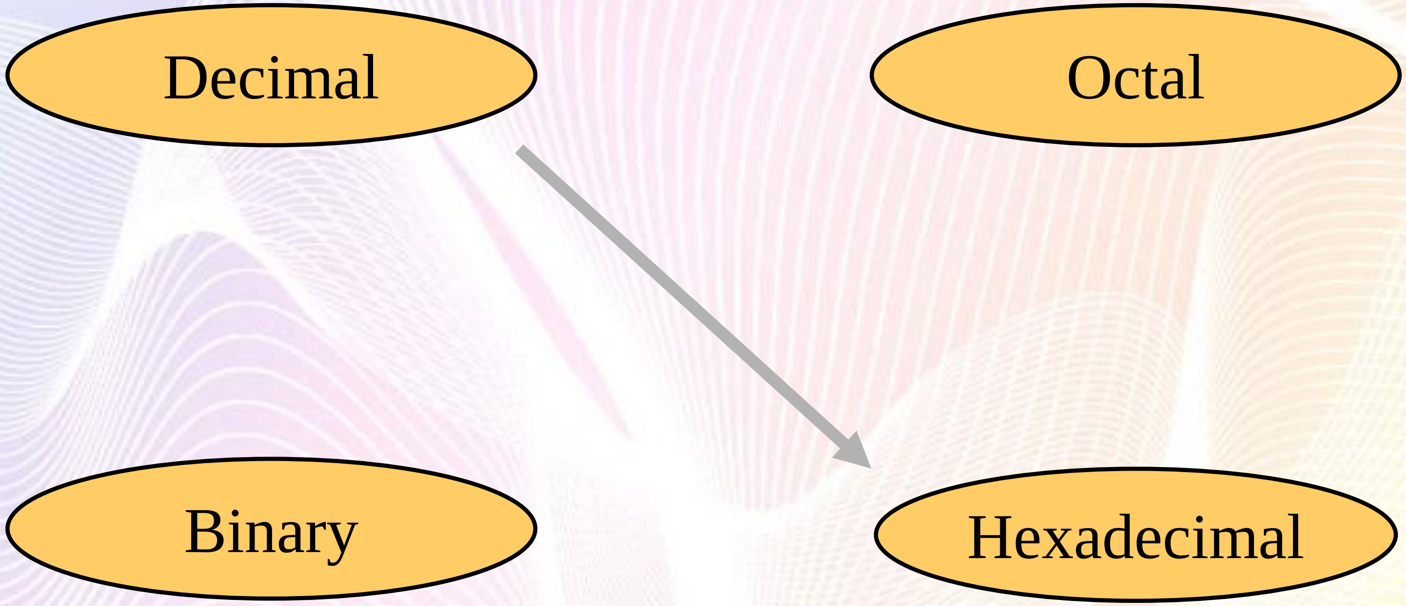
Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal



Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

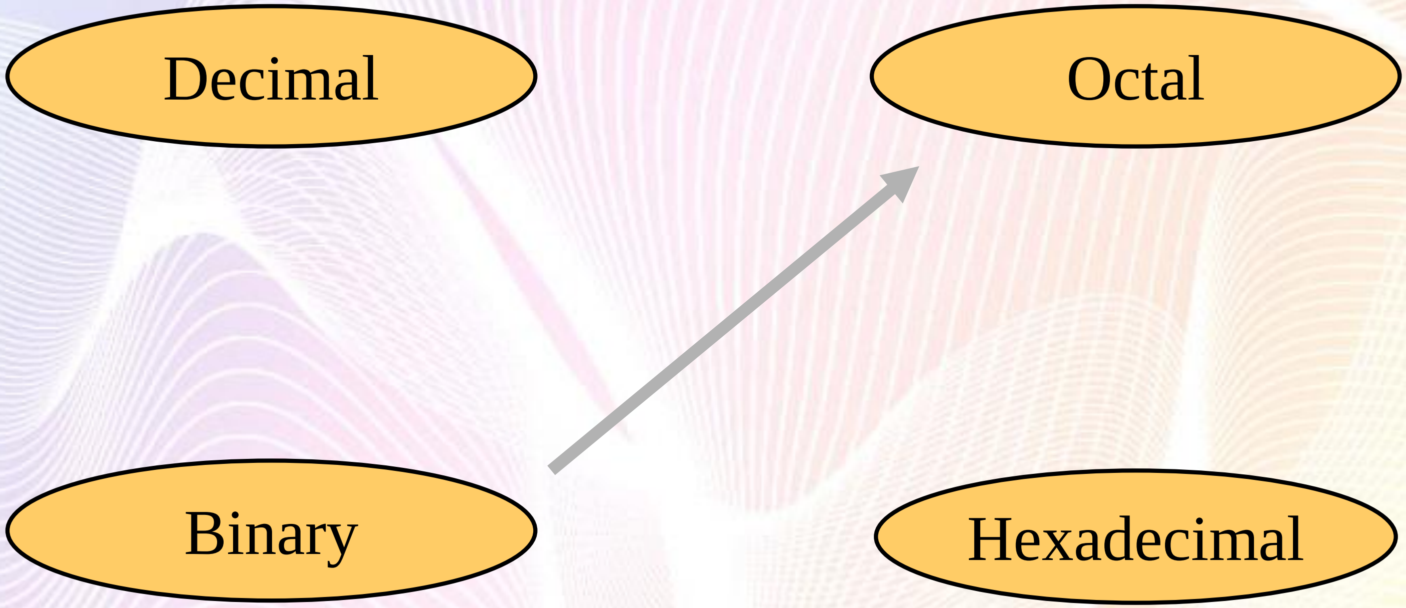
Example

$1234_{10} = ?_{16}$

```
16 | 1234
16 |   77     2
16 |    4    13 = D
        0     4
```

$1234_{10} = 4D2_{16}$

Binary to Octal



Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

Example

$1011010111_2 = ?_8$

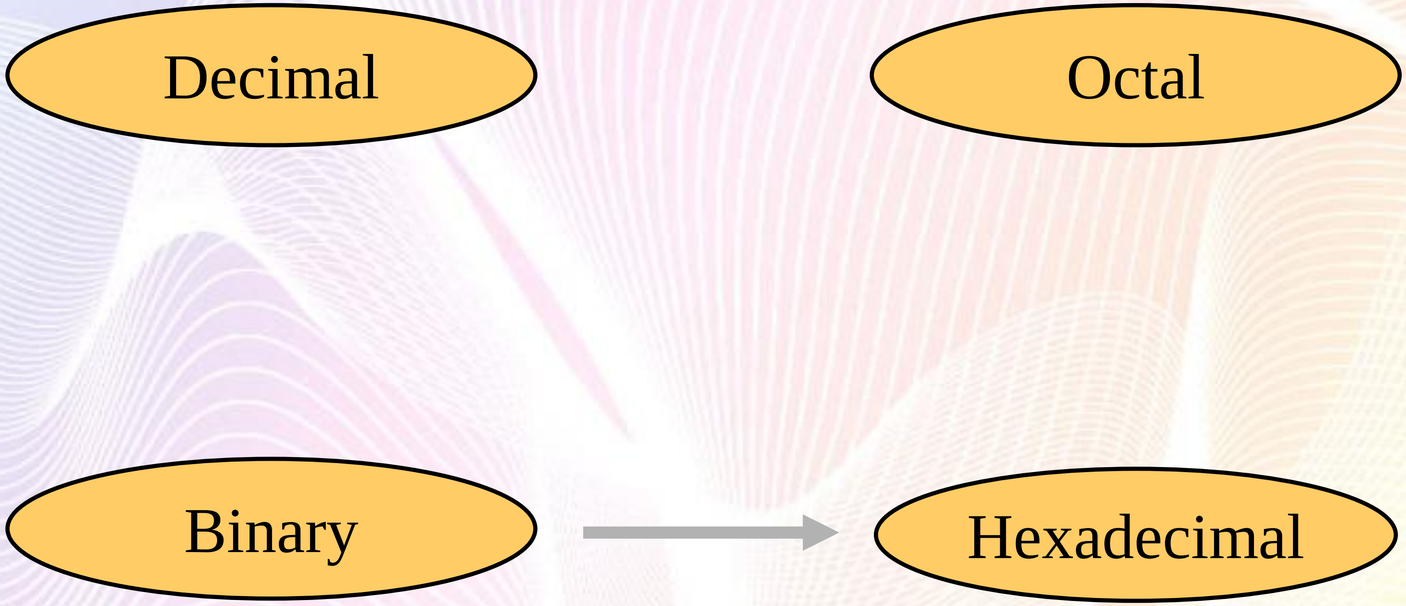
1 011 010 111

↓ ↓ ↓ ↓

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

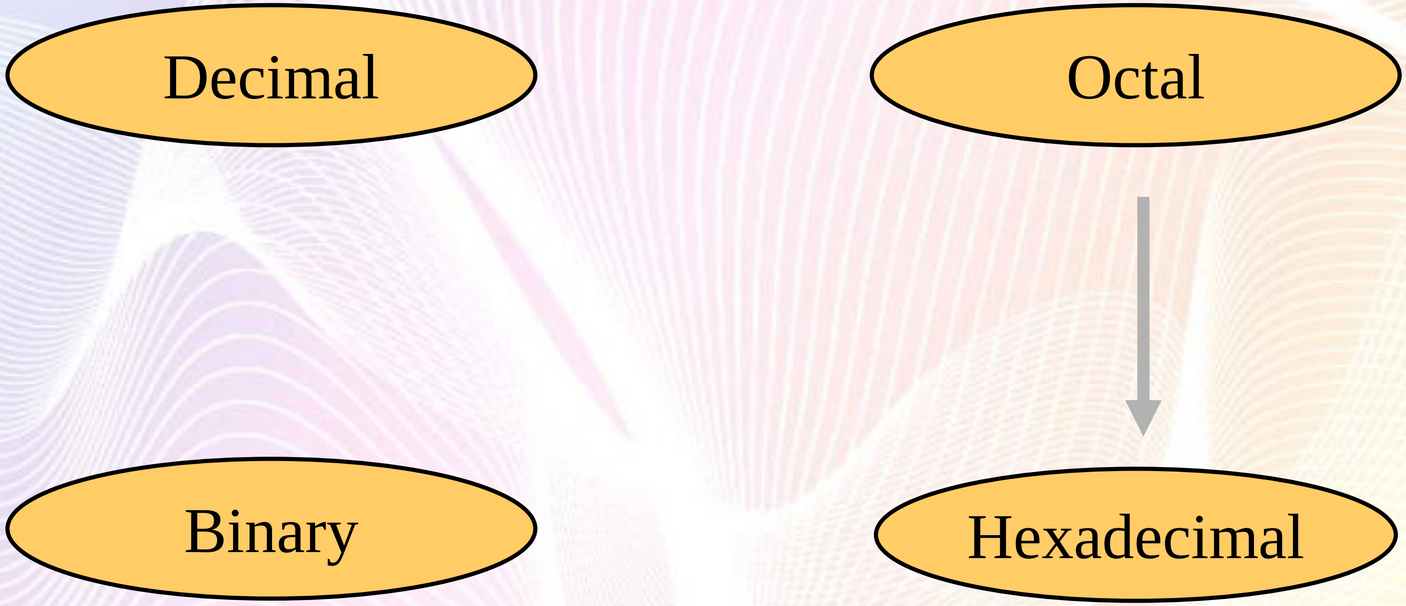
$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal



Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

Example

$1076_8 = ?_{16}$

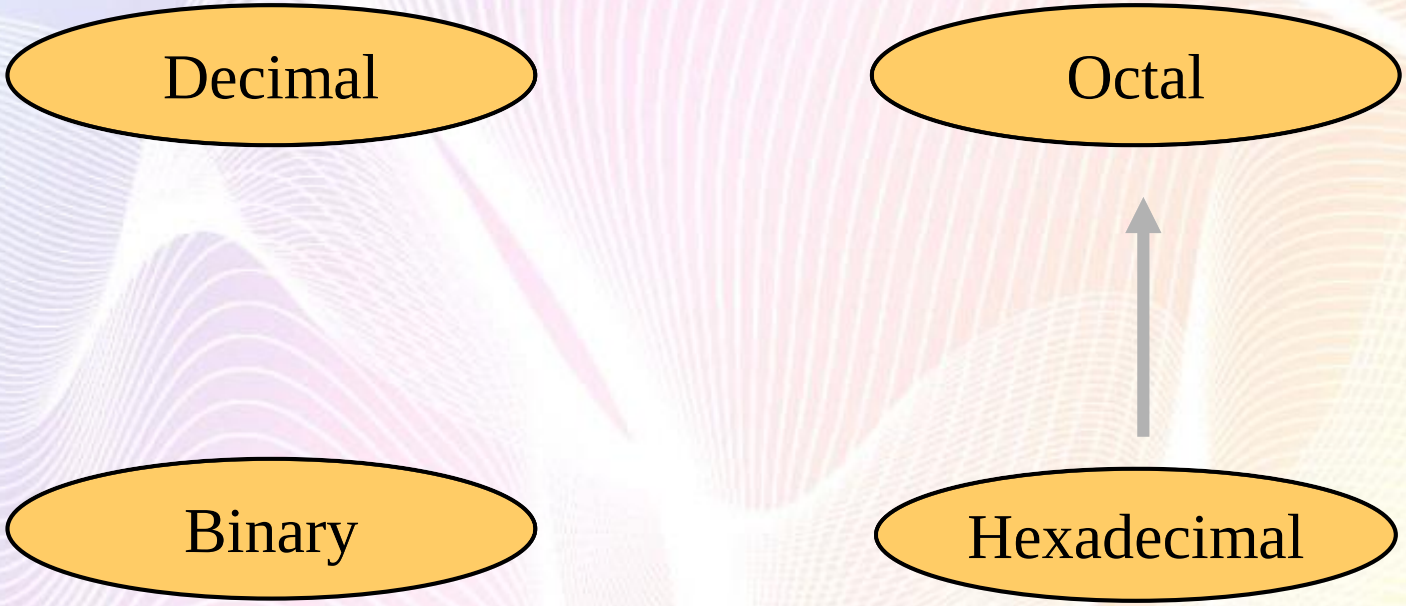
1 0 7 6

001 000 111 110

0|00 11|1 → 2 | 3 | E

$1076_8 = 23E_{16}$

Hexadecimal to Octal



Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

Example

$1F0C_{16} = ?_{8}$

1	F	0	C
0001	1111	0000	1100

0 | 001 | 111 | 1 000 | 0 1 | 100

1	7	4	1	4

$1F0C_{16} = 17414_{8}$

Binary Addition (1 of 2)

- Two 1-bit values

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

"two"

pp. 36-38

Binary Addition (2 of 2)

- Two *n*-bit values
  - Add individual bits
  - Propagate carries
  - E.g.,

```
 1   1
  10101       21
+ 11001     + 25
 101110       46
```

Multiplication (1 of 3)

- Decimal (just for fun)

```
   35
x 105
-----
  175
 000
 35
-----
 3675
```

Multiplication (2 of 3)

- Binary, two 1-bit values

A	B	$A \times B$
0	0	0
0	1	0
1	0	0
1	1	1

Multiplication (3 of 3)

- Binary, two *n*-bit values
  - As with decimal values
  - E.g.,

```
      1110
    x 1011
  --------
      1110
     1110
    0000
   1110
  --------
  10011010
```

Fractions

- Binary to decimal

10.1011 =>

$1 \times 2^{-4} = 0.0625$
$1 \times 2^{-3} = 0.125$
$0 \times 2^{-2} = 0.0$
$1 \times 2^{-1} = 0.5$
$0 \times 2^{0} = 0.0$
$1 \times 2^{1} = 2.0$

2.6875

Fractions

- Decimal to binary

3.14579

```
 .14579
x     2
-------
0.29158
x     2
-------
0.58316
x     2
-------
1.16632
x     2
-------
0.33264
x     2
-------
0.66528
x     2
-------
1.33056

etc.
```

11.001001...

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Don't use a calculator!

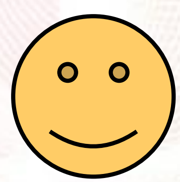
Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82



Thank you